# Economic Growth & Development: Part 3
# Horizontal Innovation Models: Applications

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# Directed Technological Change

## Technological Change: Classifications (Acemoglu; Ch.15.2)

Let $Y = \tilde{F}(K, L, A)$ is linear homogenous in $K$ and $L$, and $A$ is the state of technology. An increase in $A$ is

- **Labor-augmenting** (Harrod-neutral) if $Y = F(K, AL)$
- **Capital-augmenting** (Solow-neutral); if $Y = F(AK, L)$
- **TFP-augmenting** (Hicks-neutral); if $Y = F(AK, AL) = AF(K, L)$

These classifications should not be confused with the related, but distinct notion of factor-biased. An increase in $A$ is

- **Labor-biased** if $\dfrac{\partial}{\partial A}\left(\dfrac{r}{w}\right) = \dfrac{\partial}{\partial A}\left(\dfrac{\tilde{F}_K(K, L, A)}{\tilde{F}_L(K, L, A)}\right) < 0$
- **Capital-biased** if $\dfrac{\partial}{\partial A}\left(\dfrac{r}{w}\right) = \dfrac{\partial}{\partial A}\left(\dfrac{\tilde{F}_K(K, L, A)}{\tilde{F}_L(K, L, A)}\right) > 0$
- **Unbiased** if $\dfrac{\partial}{\partial A}\left(\dfrac{r}{w}\right) = \dfrac{\partial}{\partial A}\left(\dfrac{\tilde{F}_K(K, L, A)}{\tilde{F}_L(K, L, A)}\right) = 0$

Clearly, TFP-augmenting (Hicks-neutral) change is unbiased. But, how about labor- or capital-augmenting changes?

**CES example:**

$$Y = F(A_K K, A_L L) \equiv \left[ \gamma_K [A_K K]^{1-\frac{1}{\varepsilon}} + \gamma_L [A_L L]^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\Rightarrow \frac{r}{w} = \frac{F_k}{F_L} = \left[\frac{\gamma_K}{\gamma_L}\right]\left[\frac{A_K}{A_L}\right]^{1-\frac{1}{\varepsilon}}\left[\frac{K}{L}\right]^{-\frac{1}{\varepsilon}}.$$

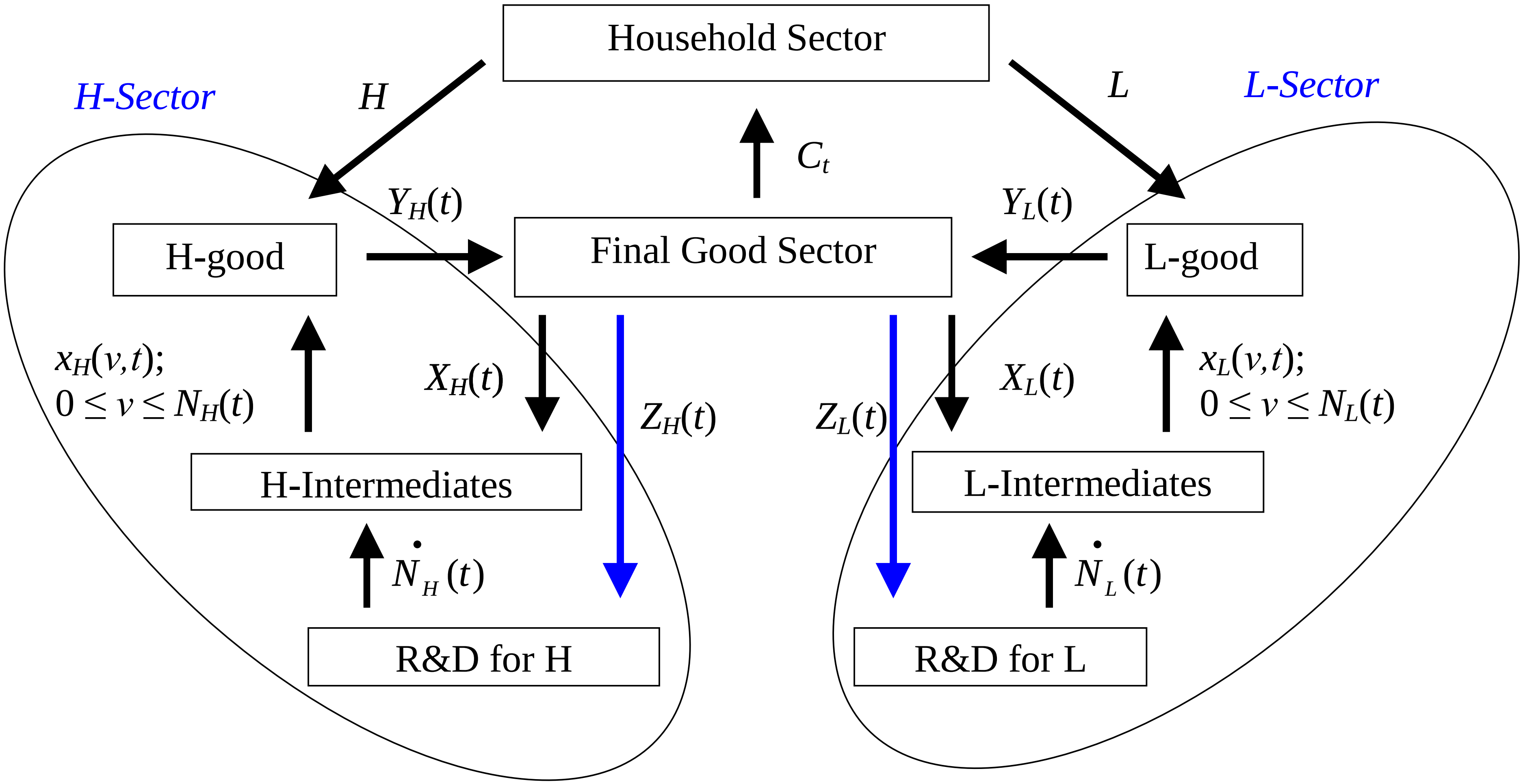
- If $\varepsilon > 1$, $K$-augmenting change is $K$-biased and $L$-augmenting change is $L$-biased.
- If $\varepsilon < 1$, $K$-augmenting change is $L$-biased and $L$-augmenting change is $K$-biased.
- If $\varepsilon = 1$, $K$-augmenting and $L$-augmenting changes are both unbiased.

Intuition for $L$-augmenting change is $K$-biased, when $\varepsilon < 1$.

An increase in labor productivity creates “capital shortage” and “excess supply of labor” at the initial factor prices. This causes a drop in the wage/rental ratio. When the scope of substitution between the two factors limited, this drop in the wage/rental ratio is more than the productivity gain, and hence the overall effect is against labor in favor of capital.

The matter is more complicated because the relative supply of factors might also affect the direction of technological change.

**Two-Sector Lab-Equipment Model (Acemoglu Ch.15.3):** Constant, inelastic supplies of two-types of labor, H & L. There are two sectors, H-sector and L-sector. The final good is produced by combining the H-good and L-good.



**Final Good Sector:** *numeraire*, competitive with CRS Technology, which combines the H-good and L-good:

$$Y(t) = F(Y_H(t), Y_L(t)) \equiv \left[\gamma_H [Y_H(t)]^{1-\frac{1}{\varepsilon}} + \gamma_L [Y_L(t)]^{1-\frac{1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

***Relative Demand for H-good/L-good:***

$$p(t) \equiv \frac{p_H(t)}{p_L(t)} = \frac{\gamma_H}{\gamma_L}\left[\frac{Y_H(t)}{Y_L(t)}\right]^{-\frac{1}{\varepsilon}} \quad \text{with} \quad p_Y(t) \equiv \left[(\gamma_H)^{\varepsilon}[p_H(t)]^{1-\varepsilon} + (\gamma_L)^{\varepsilon}[p_L(t)]^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1$$

**H-Goods and L-Goods Production:** competitive with CRS technology

$$Y_H = \frac{1}{1-\beta}\left[\int_0^{N_H} [x_H(\nu)]^{1-\beta} d\nu\right] H^{\beta}; \qquad Y_L = \frac{1}{1-\beta}\left[\int_0^{N_L} [x_L(\nu)]^{1-\beta} d\nu\right] L^{\beta}$$

- The state of technology is now two-dimension, $\{N_H, N_L\}$.
- *H* works only with *H*-intermediates in the *H*-sector.
- *L* works only with *L*-intermediates in the *L*-sector.

**Demand for an H-intermediate:**

From Max $p_H Y_H - \int_0^{N_H} p_H^x(v)x_H(v)dv - w_H H$.  $\Rightarrow x_H = H\left(\frac{p_H}{p_H^x}\right)^{1/\beta}$.

**H-Intermediate producers:** $\psi = 1-\beta$ units of the final good are required to produce one unit of each variety. Hence, $p_H^x(v,t)(1-\beta) = \psi \Rightarrow p_H^x(v,t) = 1$ for all $v$ and $t$.

$$\Rightarrow x_H(t) = H\big(p_H(t)\big)^{1/\beta}; \qquad \pi_H(t) = \beta H\big(p_H(t)\big)^{1/\beta}$$

Its scale of operation and its profit both increase with $H$ (the market size effect) and $p_H$ (the price effect).

$$\Rightarrow \frac{Y_H(t)}{HN_H(t)} = \frac{(p_H(t))^{\frac{1-\beta}{\beta}}}{1-\beta}; \qquad \frac{X_H(t)}{HN_H(t)} = (1-\beta)\big(p_H(t)\big)^{1/\beta}; \quad \frac{w_H(t)}{N_H(t)} = \frac{\beta(p_H(t))^{1/\beta}}{1-\beta}$$

- $N_H$ can be viewed as the productivity of $H$.
- $w_H$ is hence proportional to $N_H$, controlling for $p_H$ (but $p_H$ will change with $N_H$).

Similar expressions hold for $L$; Hence,

## Equilibrium Relative Output Price (in the Short Run):

Relative Supply➔ $\left(p(t)\right)^{\frac{1-\beta}{\beta}}\left(\frac{N_H(t)}{N_L(t)}\right)\left(\frac{H}{L}\right)=\frac{Y_H(t)}{Y_L(t)}=\left(\frac{\gamma_H}{\gamma_L}\right)^{\varepsilon}\left[p(t)\right]^{-\varepsilon}$ 🡐Relative Demand

$$\Rightarrow p(t)=\left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon\beta}{\sigma}}\left[\frac{N_H(t)H}{N_L(t)L}\right]^{-\frac{\beta}{\sigma}},$$

where $\sigma\equiv 1+\beta(\varepsilon-1)$. Note $\sigma>(<)1 \Leftrightarrow \varepsilon>(<)1$.

## Relative Factor Price (in the Short Run):

$$\omega(t)\equiv\frac{w_H(t)}{w_L(t)}=\frac{N_H(t)}{N_L(t)}\left(p(t)\right)^{1/\beta}=\left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}}\left[\frac{N_H(t)}{N_L(t)}\right]^{1-\frac{1}{\sigma}}\left[\frac{H}{L}\right]^{-\frac{1}{\sigma}}$$

Hence, $\sigma$ can be viewed as the (derived) elasticity of substitution between $H$ & $L$.

- $H$-augmenting change is $H$-biased if $\sigma > 1$ and $L$-biased if $\sigma < 1$.
- $L$-augmenting change is $L$-biased if $\sigma > 1$ and $H$-biased if $\sigma < 1$.

**R&D Sectors:** $\dot{N}_H = \eta_H Z_H$; $\dot{N}_L = \eta_L Z_L$, where the returns for innovation are given by

$$V_H(t) = \int_t^\infty \pi_H(s)\exp\left[-\int_t^s r(u)du\right]ds; \qquad V_L(t) = \int_t^\infty \pi_L(s)\exp\left[-\int_t^s r(u)du\right]ds.$$

Along the BGP, $r(t)$ & $p(t)$ are constant, hence so are $\pi_H(t), \pi_L(t)$, & $V_H(t), V_L(t)$.

$$\frac{V_H}{V_L} = \frac{\pi_H}{\pi_L} = \left(\frac{H}{L}\right)\left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}}$$ with its market size effect, $\left(\frac{H}{L}\right)$, & the price effect, $\left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}}$.

$$\Rightarrow \frac{V_H}{V_L} = \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}}\left[\frac{N_H(t)}{N_L(t)}\right]^{-\frac{1}{\sigma}}\left[\frac{H}{L}\right]^{1-\frac{1}{\sigma}}$$

If $\dot{N}_H > 0$ and $\dot{N}_L > 0$, $\eta_H V_H = 1 = \eta_L V_L$

$$\Rightarrow \frac{\eta_H}{\eta_L}\left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}}\left[\frac{N_H(t)}{N_L(t)}\right]^{-\frac{1}{\sigma}}\left[\frac{H}{L}\right]^{1-\frac{1}{\sigma}} = 1 \qquad \Rightarrow \frac{N_H(t)}{N_L(t)} = \left(\frac{\eta_H}{\eta_L}\right)^{\sigma}\left(\frac{\gamma_H}{\gamma_L}\right)^{\varepsilon}\left[\frac{H}{L}\right]^{\sigma-1}$$

- A higher *H/L* cause *H*-augmenting change if $\sigma > 1$ and *L*-augmenting change if $\sigma < 1$.

## Relative Factor Price (in the Long Run):

$$\omega(t) = \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}} \left[\frac{N_H(t)}{N_L(t)}\right]^{1-\frac{1}{\sigma}} \left[\frac{H}{L}\right]^{-\frac{1}{\sigma}}$$

Along the BGP,

$$\omega = \left(\frac{\eta_H}{\eta_L}\right)^{\sigma-1} \left(\frac{\gamma_H}{\gamma_L}\right)^{\varepsilon} \left[\frac{H}{L}\right]^{\frac{(\sigma-1)^2-1}{\sigma}} = \left(\frac{\eta_H}{\eta_L}\right)^{\sigma-1} \left(\frac{\gamma_H}{\gamma_L}\right)^{\varepsilon} \left[\frac{H}{L}\right]^{\sigma-2}.$$

The relative demand curve is flatter in the long run than in the short-run ($\sigma - 2 > -1/\sigma$ whenever $\sigma \neq 1$), because a higher *H/L* causes *H*-biased technological change, which shifts the demand curve outward.

**Intuition:**

1) When *H* & *L* are close substitutes ($\sigma > 1$),
- A higher *H/L* causes *H*-augmenting change;
- *H*-augmenting change is *H*-biased;

Hence, it is H-biased.

2) When *H* & *L* are not so close substitutes ($\sigma < 1$),
- A higher *H/L* causes *L*-augmenting change;
- *L*-augmenting change is *H*-biased;

Hence, it is H-biased.

Either way, a higher *H/L* induces H-biased technological change. (**Weak Equilibrium Bias** in Acemoglu's terminology)

Also, if $\sigma > 2$, Bias is so strong that the relative demand becomes upward-sloping. (**Strong Equilibrium Bias** in Acemoglu's terminology)

Acemoglu (Ch.15) argue that these results help to understand:

- Unskill-biased technological change in the 19th century and more skill-biased technological change in the 20th century;
- Temporary decline in the college premium in the seventies and subsequent rise in the premium in the eighties. (This, however, requires $\sigma > 2$.)

More general lesson is that a change in the relative supply of skills not only responds to biased technological changes, but also could cause biased technological changes.

**Exercise:** Complete the analysis by finding the conditions ensuring the existence of BGP.

**Note on Transition Dynamics:** If $N_H(0)/N_L(0) < (N_H/N_L)^*$, $\dot{N}_H > 0 = \dot{N}_L$ until it reaches the BGP. Likewise, if $N_H(0)/N_L(0) > (N_H/N_L)^*$, $\dot{N}_H = 0 < \dot{N}_L$ until it reaches the BGP.

**Two-Sector Knowledge-Spillover Model (Acemoglu, Ch.15.4):**

Modify the above model by:

- Households have a constant supply of *H, L,* and *S.*
- *H* and *L* are used only in *H*- and *L*-sectors as before.
- Scientists (*S*) are used only in the R&D sector, as its only inputs (instead of the final good.) The R&D technologies are

$$\dot{N}_H = \eta_H (N_H)^{\frac{1+\delta}{2}} (N_L)^{\frac{1-\delta}{2}} S_H ; \dot{N}_L = \eta_L (N_L)^{\frac{1+\delta}{2}} (N_H)^{\frac{1-\delta}{2}} S_L ; \qquad S_L + S_H = S .$$

where $\delta$ introduces the persistence in the direction of R&D. Along the BGP,

$$\eta_H (N_H)^{\frac{1+\delta}{2}} (N_L)^{\frac{1-\delta}{2}} V_H = w_S = \eta_L (N_L)^{\frac{1+\delta}{2}} (N_H)^{\frac{1-\delta}{2}} V_L$$

$$\Rightarrow \eta_H (N_H)^{\delta} V_H = \eta_L (N_L)^{\delta} V_L$$

$$\Rightarrow \left(\frac{\eta_H}{\eta_L}\right)^{-1} \left[\frac{N_H(t)}{N_L(t)}\right]^{-\delta} = \frac{V_H}{V_L} = \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}} \left[\frac{N_H(t)}{N_L(t)}\right]^{-\frac{1}{\sigma}} \left[\frac{H}{L}\right]^{1-\frac{1}{\sigma}}$$

$$\Rightarrow \frac{N_H(t)}{N_L(t)} = \left(\frac{\eta_H}{\eta_L}\right)^{\frac{\sigma}{1-\sigma\delta}} \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{1-\sigma\delta}} \left[\frac{H}{L}\right]^{\frac{\sigma-1}{1-\sigma\delta}} .$$

*Note:* Convergence towards BGP requires $\sigma\delta < 1$. Too much persistence and/or substitution leads to instability.

**Relative Factor Prices:**

$$\frac{w_H(t)}{w_L(t)} = \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}} \left[\frac{N_H(t)}{N_L(t)}\right]^{1-\frac{1}{\sigma}} \left[\frac{H}{L}\right]^{-\frac{1}{\sigma}} \propto \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}},$$

upward-sloping for $\sigma > 2 - \delta$, under the condition less stringent than before.

**Growth Rate:**

$$\dot{N}_H = \eta_H (N_H)^{\frac{1+\delta}{2}} (N_L)^{\frac{1-\delta}{2}} S_H; \quad \dot{N}_L = \eta_L (N_L)^{\frac{1+\delta}{2}} (N_H)^{\frac{1-\delta}{2}} S_L; \qquad S_L + S_H = S.$$

$$\Rightarrow \eta_H (N_H)^{-\frac{1-\delta}{2}} (N_L)^{\frac{1-\delta}{2}} S_H = \frac{\dot{N}_H}{N_H} = g^* = \frac{\dot{N}_L}{N_L} = \eta_L (N_H)^{\frac{1-\delta}{2}} (N_L)^{-\frac{1-\delta}{2}} (S - S_H)$$

$$\Rightarrow g^* = \left[\frac{1}{\eta_H}\left(\frac{N_H}{N_L}\right)^{\frac{1-\delta}{2}} + \frac{1}{\eta_L}\left(\frac{N_H}{N_L}\right)^{-\frac{1-\delta}{2}}\right]^{-1} S$$

***Notes:***

- The existence of BGP is ensured by imposing restrictions on parameters, $\theta$ and $\rho$.
- The growth rate is maximized at $\frac{N_H}{N_L} = \left(\frac{\eta_H}{\eta_L}\right)^{\frac{1}{1-\delta}}$, independent of $\sigma$ or $\gamma_H/\gamma_L$.

**Knowledge-Spillover Case without "Scale Effect" (Acemoglu, Ch.15.5)**

Now, change the above model such that

- Households have an inelastic supply of *H, L,* & *S,* growing at the rate, *n.*
- R&D technology is given by

$$\dot{N}_H = \eta_H (N_H)^{\lambda} S_H; \qquad \dot{N}_L = \eta_L (N_L)^{\lambda} S_L; \qquad S_L + S_H = S;$$

Along the BGP, $\eta_H (N_H)^{\lambda} V_H = w_S = \eta_L (N_L)^{\lambda} V_L$

$$\Rightarrow \left(\frac{\eta_H}{\eta_L}\right)^{-1}\left[\frac{N_H(t)}{N_L(t)}\right]^{-\lambda} = \frac{V_H}{V_L} = \left(\frac{\gamma_H}{\gamma_L}\right)^{\frac{\varepsilon}{\sigma}}\left[\frac{N_H(t)}{N_L(t)}\right]^{-\frac{1}{\sigma}}\left[\frac{H}{L}\right]^{1-\frac{1}{\sigma}}$$

$$\Rightarrow \frac{N_H}{N_L} \propto \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\lambda\sigma}}$$

$$\Rightarrow \frac{w_H(t)}{w_L(t)} \propto \left(\frac{H}{L}\right)^{\frac{\sigma-2+\lambda}{1-\lambda\sigma}}$$

, upward sloping if $\sigma > 2 - \lambda$, less stringent condition than before.

## *L*-Augmenting Technological Change and (Endogenously) Constant Factor Shares (Acemoglu, Ch.15.6)

**Sketch of the idea** (not very rigorous): Knowledge-spillover models with $L$, and, instead of $H$, $K(t)$, which grows at a constant rate.

$$\frac{N_K(t)}{N_L(t)} \propto \left[\frac{K(t)}{L}\right]^{\frac{\sigma-1}{1-\delta\sigma}}; \qquad \frac{r(t)}{w(t)} \propto \left(\frac{K(t)}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}$$

With $\sigma < 1$,

- $N_k/N_L$ declines as $K(t)/L$ grows. ➔ More $L$-augmenting (closer to Harrod-neutral). But, *not purely* L-augmenting and no BGP.
- With $\delta = 1$, $\dfrac{r(t)}{w(t)} \propto \left(\dfrac{K(t)}{L}\right)^{-1}$.

➢$rK(t)/(w(t)L)$ is constant asymptotically.

➢BGP with purely $L$-augmenting *(i.e.,* Harrod-neutral), which is globally stable.

**Intuition:** With $\sigma < 1$, capital accumulation makes the price of labor and profits from labor-augmenting technologies rise sufficiently faster, which causes the Harrod-neutral change, keeping the effective supplies of the two factors in balance.

# Development Traps (Ciccone and Matsuyama)

# Endogenous Cycles (Matsuyama 1999)